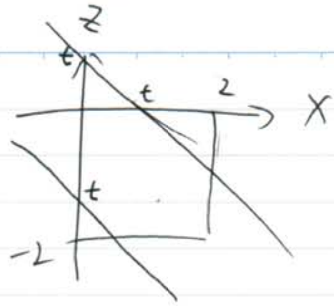
(6) $T = X + Z$



$P(X+Z<t)$

$= P(Z < t - X)$

$= \begin{cases} \frac{1}{4}\cdot\frac{1}{2}(2+t)^2 & (-2\le t\le 0) \\ \frac{1}{4}\left(4-(2-t)^2\cdot\frac{1}{2}\right) & (0\le t\le 2) \end{cases}$

$\longrightarrow f_T(t) = \begin{cases} \frac{1}{4}(2+t) & -2\le t\le 0 \\ \frac{1}{4}(2-t) & 0< t\le 2 \end{cases}$

$S = T + Y \quad V = T \quad \rightarrow T = V,\ Y = S - V$

$f_{S,V}(s,v)\,ds\,dv = f_{T,Y}(t,y)\,dt\,dy$

$\left\| \frac{dt\,dy}{ds\,dv} \right\| = \left\| \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix} \right\| \rightarrow 1$

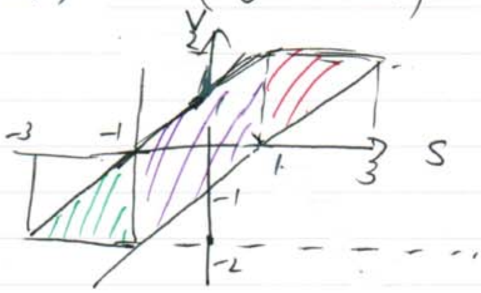
$f_{S,V}(s,v) = f_T(t)\cdot f_Y(y)\cdot 1 = f_T(v)\cdot\frac{1}{2}$

$= \begin{cases} \frac{1}{8}(2+v) & (-2\le v\le 0) \\ \frac{1}{8}(2-v) & (0\le v\le 2) \end{cases}$

↓ Sの周辺分布をもとめたい. 積分区間

$-2\le V\le 2$

$-1\le S-V\le 1$



$-3\le S\le 1 \rightarrow f_S = \int_{-2}^{1+s}\frac{1}{8}(2+v)\,dv = \frac{1}{16}\left[(2+v)^2\right]_{-2}^{1+s} = \frac{1}{16}(3+s)^2$

$-1< S< 1 \rightarrow f_S = \int_{s-1}^{0}\frac{1}{8}(2+v)\,dv + \int_{0}^{s+1}\frac{1}{8}(2-v)\,dv$

$= \frac{1}{16}\left[(2+v)^2\right]_{s-1}^{0} - \frac{1}{16}\left[(2-v)^2\right]_{0}^{s+1} = \frac{1}{8}(3-s^2)$

$1\le S\le 3 \quad f_S = \int_{s-1}^{2}\frac{1}{8}(2-v)\,dv = \frac{1}{16}\left[-(2-v)^2\right]_{s-1}^{2} = \frac{1}{16}(3-s)^2$